Fiscal Policy in Small Open Economies: The International Intertemporal Keynesian Cross*

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Abstract

A recent literature studies fiscal policy with realistic intertemporal marginal propensities to consume and the absence of Ricardian equivalence. This literature shows that fiscal policy boosts consumption and is effective: (i) Cumulative fiscal multipliers are above one and (ii) fiscal policy is fully self-financing. I show that these results do not extend to small open economies (SOEs). In SOEs, the initial debt-fueled rise in consumption is offset by a subsequent drop to pay back foreign debt, so (i) the cumulative fiscal multiplier is exactly one, and (ii) fiscal deficits are not fully self-financing.

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1 Introduction

A recent literature studies fiscal policy in the absence of Ricardian equivalence. They find that with empirically realistic intertemporal marginal propensities to consume (iMPCs), increased government consumption causes consumption to increase, boosting output and fiscal multipliers. Two central results for closed economies with a fixed real interest rate are:

1. The cumulative fiscal multiplier is greater than one (Auclert et al. 2024).
2. Fiscal deficits are fully self-financing (Angeletos et al. 2024).

I re-visit these results in a small open economy (SOE) using an International Intertemporal Keynesian Cross (IIKC) with passive monetary policy, which nests the closed economy of Auclert et al. (2024) when the degree of openness is zero. I find starkly different results when the degree of openness is strictly positive:

1. The cumulative fiscal multiplier is exactly one.
2. Fiscal deficits are less than fully self-financing.

Why is fiscal policy less effective in an SOE? As in the closed economy, expansionary fiscal policy initially generates a rise in consumption. Thus, there is initial crowding-in and the impact fiscal multiplier is above one. The rise in consumption is financed by borrowing from abroad. This current account deficit has to be repaid, which households do by cutting consumption. I show analytically that the drop in consumption exactly offsets the initial rise, such that the consumption is unchanged in cumulative present value terms. This implies a unit cumulative fiscal multiplier.

The zero response of consumption in present value terms and the unit cumulative fiscal multiplier are robust: They hold for (i) any path of government consumption,

(ii) any timing of financing, (iii) any degree of openness, and (iv) any household behavior satisfying a standard budget constraint and transversality condition.[1]

This has implications for our understanding of the determinants of fiscal multipliers. For instance, the path of primary deficits plays no role in determining the cumulative fiscal multiplier in the SOE, while they play a central role in the closed economy. Additionally, the cumulative fiscal multiplier is the same for a heterogeneous agent model (incomplete markets) and a representative agent model (complete markets) in an SOE, while they are drastically different in a closed economy.

Lastly, I explore some extensions where the cumulative multiplier is not always one. This is possible (i) when looking at the cumulative fiscal multiplier over only a few years, (ii) when the domestic economy is not infinitesimal, and (iii) with active monetary policy.

Related Literature Firstly, my paper contributes to the literature on fiscal policy in Heterogeneous Agent New Keynesian (HANK) models. Apart from Auclert et al. (2024) and Angeletos et al. (2024), analytical results for closed economy have in particular been provided by Challe and Ragot (2010), Acharya and Dogra (2020), Bilbiie (2021), and Broer et al. (2021). I differ by focusing on an SOE. Hagedorn et al. (2019) provides a quantitative analysis of fiscal policy in HANK models for a closed economy. No similar analysis has been done for an SOE.[2] My analytical results provide the basis for such an exploration.

Secondly, my paper contributes to the growing literature on open economy HANK models, see e.g. Ferra et al. (2020), Auclert et al. (2021c), Guo et al. (2023), Oskolkov (2023) and Druedahl et al. (2024). My analytical results for fiscal policy are unique.

1. This includes a household side without and with—any degree of—heterogeneity.

2. Aggarwal et al. (2023) study fiscal deficits in a multi-country HANK model, including SOEs.

Structure I split my paper into two parts. In Section 2, I present a standard model of an SOE with a generalized household side and use it to derive an International Intertemporal Keynesian Cross. In section 3, I study the effectiveness of fiscal policy in this setup by deriving analytical results for the fiscal multiplier and degree of self-financing. Finally, I conclude in Section 4.

2 A Simple Small Open Economy

2.1 The Model

In this section, I present a simple model of a small open economy. The model consists of two parts: The household side and the small open economy New-Keynesian (SOE-NK) part. The SOE-NK part follows the textbook model from Gali and Monacelli (2005). On the other hand, the household side of the model is general, featuring minimal assumptions. This general structure nests standard representative agent and heterogeneous agent models. Having written the full model, I show how it admits an *International* Intertemporal Keynesian Cross (IIKC), which I use to study fiscal policy.[3] This IIKC nests the Intertemporal Keynesian Cross (IKC) from Auclert et al. 2024 when the economy is closed.

2.1.1 Households

I start by describing the household side. At each time $t = 0, 1, \ldots,$ households' aggregate budget constraint is

$$C_t + A_t = (1 + r)A_{t-1} + Z_t, \tag{1}$$

3. Auclert et al. (2021c) study a similar IIKC.

where C_t is consumption, A_t is savings, $r > 0$ is the real interest rate, and Z_t is real disposable labor income. Furthermore, household behavior satisfies a standard transversality constraint:

$$\lim_{t \to \infty} \frac{A_t}{(1+r)^t} = 0. \tag{2}$$

Aggregate consumption is given by the sequence space consumption function

$$C_t = \mathcal{C}_t \left(\{Z_s\}_{s=0}^{\infty} \right). \tag{3}$$

This general household problem nests a broad class of specific household problems. Before proceeding with the rest of the model, let me briefly sketch some standard household sides that satisfy the setup. The first example is a standard heterogeneous agents model. There is a continuum of households. Each household with assets a, and idiosyncratic earnings e chooses consumption c, and next-period assets a' (where primes denote variables in the next period). They choose this to solve the following dynamic problem:

$$\begin{aligned} V_t(a, e) &= \max_{c, a'} u(c) + \beta \mathbb{E}_t \left[V_{t+1}(a', e') \right], \\ &\text{s.t.} \\ c + a' &= (1+r)a + Z_t e, \\ a' &\geq 0, \end{aligned}$$

where u is the instantaneous utility of consumption. The individual budget constraints aggregate up to the aggregate budget constraint in eq. (1) and the model satisfies the transversality condition in eq. (2), so the model fits in my setup.

In addition to the heterogeneous agents model, the standard representative agent model also fits my setup. This is obtained simply by setting $e = 1$ and removing the borrowing constraint. Solving this problem yields a standard Euler equation:[4]

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1}). \tag{4}$$

In addition to standard Representative Agent New Keynesian (RANK) and Heterogeneous Agent New Keynesian (HANK) models, the setup presented here also nests different models like models deviating from full information, rational expectations, or even optimizing behavior.

Having set up the household problem, I round out with the definition of some aggregate variables. Aggregate real disposable labor income is

$$Z_t = w_t N_t - T_t, \tag{5}$$

where $w_t = W_t/P_t$ is the real wage, W_t is the nominal wage, P_t is the consumer price index (CPI), and T_t are taxes. Labor supply, N_t, is set by unions. Due to the fixed real interest rate, this only matters for nominal variables and not real variables, so I do not describe it here.[5]

Consumption is split across home goods, $C_{H,t}$, and foreign goods, $C_{F,t}$:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{6}$$

4. Note that the RANK model discussed here is not stationary, a well-known issue discussed in Schmitt-Grohé and Uribe (2003). However, the RANK model does satisfy $\lim_{t\to\infty} dY_t = \lim_{t\to\infty} dC_t = \lim_{t\to\infty} dG_t = \lim_{t\to\infty} dT_t = 0$, so computing fiscal multipliers and analyzing the IIKC is well-defined. As discussed in Auclert et al. (2021c), the HANK model is stationary due to precautionary savings.

5. See Appendix A.1.3.

where $P_{H,t}$ and $P_{F,t}$ are the prices of the goods. $\eta > 0$ is the elasticity of substitution, and $\alpha \in [0,1]$ is the degree of openness. The CPI is then

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{7}$$

2.1.2 The Government

The government issues real bonds, B_t, and runs a primary deficit, PD_t, such that its real budget constraint is

$$B_t = (1+r)B_{t-1} + \mathrm{PD}_t. \tag{8}$$

The primary deficit is $\mathrm{PD}_t = (P_{H,t}/P_t)G_t - T_t$, where G_t is government consumption. The transversality condition for the government is $\lim_{t\to\infty} B_t/(1+r)^t = 0$.

2.1.3 Production

The supply side is simple. Production has constant returns to scale in labor:

$$Y_t = N_t.$$

The price is set at the marginal cost, $P_{H,t} = W_t$. When foreign goods are sold abroad, their price in foreign currency is set using the nominal exchange rate, E_t, i.e. $P_{H,t}^* = P_{H,t}/E_t$. Domestic production is consumed by domestic households, foreign households, and the government:

$$Y_t = C_{H,t} + C_{H,t}^* + G_t. \tag{9}$$

2.1.4 The Foreign Economy

The home country trades with a foreign economy. The consumption of home goods by foreign consumers is then

$$C^*_{H,t} = \alpha \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} C^*_t, \tag{10}$$

where C^*_t is consumption abroad and P^*_t is the foreign CPI. The real exchange rate is

$$Q_t = \frac{E_t P^*_t}{P_t}, \tag{11}$$

where $E_t = P_{F,t}/P^*_{F,t}$ and $P^*_{F,t}$ is the price of foreign goods abroad.

Free capital flows and arbitrage across countries imply a standard uncovered interest rate parity (UIP) condition:

$$1 + r = (1 + r^*_t) \frac{Q_{t+1}}{Q_t}, \tag{12}$$

where r^*_t is the foreign real interest rate.

The assumption of a small open economy means that the foreign economy does not respond to fiscal policy in the SOE.[6] This implies that

$$\begin{aligned} P^*_{F,t} &= P^*_t = 1, \\ C^*_t &= C^*_{ss}, \\ r^*_t &= r^*_{ss}, \end{aligned}$$

where "ss" denotes the steady state.

6. Except for the SOE changing the price of foreign goods, $P^*_{H,t}$, which the foreign demand for domestic goods, $C^*_{H,t}$, responds to.

2.2 The International Intertemporal Keynesian Cross

I now show that the model in Section 2.1 can be represented by an IIKC. This nests the closed economy IKC of Auclert et al. (2024) for $\alpha = 0$. I derive the IIKC by linearizing the model around the steady state and writing it in the sequence space, following the seminal contribution of Auclert et al. (2021b). The key objects are sequences of deviations, $dX_t \approx X_t - X_{ss}$, from steady state, X_{ss}, of any variable X: $d\boldsymbol{X} = (dX_0, dX_1, \ldots)'$.

To derive the IIKC, I start by considering UIP in eq. (12). Since returns do not vary across countries, UIP implies that the real exchange rate does not respond to fiscal policy.[7] This implies that relative prices are fixed.[8] Therefore, real labor income is given by $Z_t = Y_t - T_t$, domestic consumption of domestic goods is $C_{H,t} = (1-\alpha)C_t$, and foreign consumption of domestic goods does not react to fiscal policy in the SOE, $C^*_{H,t} = C^*_{H,ss}$. With this, I linearize the consumption function in eq. (3):

$$d\boldsymbol{C} = \boldsymbol{M} d\boldsymbol{Z} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}),$$

where $\boldsymbol{M} \equiv \partial C/\partial Z$ is the iMPC matrix.[9] Using $dC_{H,t} = (1-\alpha)dC_t$ then gives

$$d\boldsymbol{C_H} = (1-\alpha)d\boldsymbol{C} = (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}).$$

7. I consider this as a baseline to be as close as possible to Auclert et al. (2024). Furthermore, there is substantial disagreement in the literature on the effects of fiscal policy on the real exchange rate: Several papers find either appreciations or depreciations, as I discuss in Section 3.2.3, where I consider the case with active monetary policy and hence a non-constant real exchange rate.

8. To see this, insert $Q_t = Q_{ss}$ into the definition of the real exchange rate in eq. (11) to get $E_t = P_t$. The definition of the nominal exchange rate then implies that $E_t = P_{F,t}$, so $P_t = P_{F,t}$. Inserting this into the CPI in eq. (7) and solving for $P_{H,t}$ yields $P_{H,t} = P_t$, so also $P^*_{H,t} = P^*_t$.

9. Assuming the consumption function can be linearized, as is standard, see Auclert et al. (2024).

Inserting this and $d\boldsymbol{C}_{\boldsymbol{H}}^{*} = \boldsymbol{0}$ into goods market clearing gives

$$d\boldsymbol{Y} = (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + d\boldsymbol{G}. \tag{13}$$

This is the IIKC, nesting the closed economy IKC from Auclert et al. (2024) when $\alpha = 0$. With the IIKC in hand, I am almost ready to derive analytical results on the effectiveness of fiscal policy. To do this, I start by solving the IIKC. However, it is not guaranteed that there exists a solution to the IIKC. And even if there is a solution, it is not guaranteed to be unique. For this reason, I proceed by establishing the determinacy properties of the IIKC. To do this, I first provide a key lemma: The present-value MPC is 1, as shown in Auclert et al. (2024). This lemma is central for two reasons: To establish determinacy and to provide intuition for the results on the effectiveness of fiscal policy.

Lemma 1. *The present-value MPC is* 1*:*

$$\sum_{t=0}^{\infty} \frac{MPC_{t,s}}{(1+r)^{t-s}} = 1, \tag{14}$$

where $MPC_{t,s} \equiv \partial C_t / \partial Z_s$.

Proof. I start by iterating on the aggregate budget constraint in eq. (1) and use the transversality condition to get that

$$\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)A_{ss} + \sum_{t=0}^{\infty} \frac{Z_t}{(1+r)^t}.$$

Taking the derivative w.r.t. Z_s yields

$$\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^t} = \frac{1}{(1+r)^s}.$$

Dividing by $(1+r)^s$ and using the definition of $\boldsymbol{M}$ yields eq. (14). On vector form, this can be written $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$, where $\boldsymbol{q}' = (q_0, q_1, \dots)' \equiv (1, (1+r)^{-1}, (1+r)^{-2}, \dots)$. $\square$

Intuitively, this means that any increase in household income has to be spent at some point in time in a present value sense. I now use this lemma to establish determinacy of the IIKC, which is key for using the IIKC to study the effectiveness of fiscal policy in the SOE.

Proposition 1 (Determinacy)**.** *If $\boldsymbol{M}$ is non-negative, the SOE ($0 < \alpha < 1$) is determinate, i.e. there always exists a unique solution for $d\boldsymbol{Y}$.*

Proof. Re-arrange the IIKC in eq. (13) to $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]\, d\boldsymbol{Y} = d\boldsymbol{G} - (1-\alpha)\boldsymbol{M} d\boldsymbol{T}$. Determinacy then amounts to showing that $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]$ is bijective. This follows from the operator norm of this linear map being smaller than 1. To show that this is the case, consider the norm $||x|| = \sum_{t=0}^{\infty} q_t |x_t| < \infty$. Then,

$$
\begin{aligned}
||\boldsymbol{M} d\boldsymbol{Y}|| &= \sum_{t=0}^{\infty} q_t |(\boldsymbol{M} d\boldsymbol{Y})_t| = \sum_{t=0}^{\infty} q_t \left| \sum_{s=0}^{\infty} M_{ts} dY_s \right| \\
&\leq \sum_{t=0}^{\infty} q_t \sum_{s=0}^{\infty} M_{ts} |dY_s| = \sum_{s=0}^{\infty} |dY_s| \sum_{t=0}^{\infty} q_t M_{ts} = \sum_{s=0}^{\infty} q_s |dY_s| = ||d\boldsymbol{Y}||,
\end{aligned}
$$

using the triangle inequality and $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ from Lemma 1. Thus, the operator norm of $\boldsymbol{M}$ is less than or equal to 1: $||\boldsymbol{M}|| \leq 1$. Consider now a present-value-summable $d\boldsymbol{Y}$ with $dY_t \geq 0$. In this case, the inequality is replaced by an equality, so $||\boldsymbol{M} d\boldsymbol{Y}|| = ||d\boldsymbol{Y}||$ in this particular case. This implies that $||\boldsymbol{M}|| = 1$. It then follows that $||(1-\alpha)\boldsymbol{M}|| = (1-\alpha)||\boldsymbol{M}|| = 1 - \alpha \in (0, 1)$. Thus, the operator norm of $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]$ is less than 1, so $\boldsymbol{I} - (1-\alpha)\boldsymbol{M}$ is bijective, implying determinacy. $\square$

The fact that a unique solution exists in the SOE is an attractive property, as one avoids having to pick a particular solution. This is not generically the case in a closed economy. As an example, it is well-known that a RANK model violating the Taylor principle has infinitely many solutions. Intuitively, the uniqueness in the SOE stems from the fact that a fraction α of consumption goes abroad.

In addition to the solution being unique in an SOE, it is also straightforward to obtain it. In particular, the unique solution simply follows by inverting the IIKC:

$$d\boldsymbol{Y} = [I - (1-\alpha)\boldsymbol{M}]^{-1}\left[d\boldsymbol{G} - (1-\alpha)\boldsymbol{M} d\boldsymbol{T}\right].$$

This is possible in the SOE because $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]^{-1}$ exists, while it is not possible in the closed economy because $[\boldsymbol{I} - \boldsymbol{M}]^{-1}$ does *not* exist.[10] Furthermore, this inverse has a well-defined infinite representation

$$[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]^{-1} = \boldsymbol{I} + (1-\alpha)\boldsymbol{M} + (1-\alpha)^2\boldsymbol{M}^2 + \dots$$

This representation shows the Keynesian cross logic: An increase in government consumption increases output directly, which is income for domestic households, who spend a fraction $(1-\alpha)\boldsymbol{M}$ on domestic goods, starting the next round of the Keynesian multiplier. Such a representation does *not* hold in the closed economy.

3 The Effects of Fiscal Policy

3.1 Fiscal Multipliers

With the IIKC, I now turn my attention to cumulative fiscal multipliers. Following Ramey (2016), Mountford and Uhlig (2009), Auclert et al. (2024), and others, I compute the cumulative fiscal multiplier following a government consumption shock as the cumulative present value change in output per unit of cumulative present value

10. I note that in the heterogenous agents (HA) model of Auclert et al. (2024), there does exist a unique solution with $\lim_{t\to\infty} dY_t = 0$. This is the solution I focus on when comparing the SOE to the closed economy.

change in government consumption:

$$\mathcal{M} \equiv \frac{\sum_{t=0}^{\infty} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t}} = \frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}}. \tag{15}$$

With this definition, I state a key result.

Proposition 2 (Cumulative fiscal multipliers)**.** *The cumulative fiscal multiplier is exactly* 1*,* $\mathcal{M} = 1$*, in an SOE* ($0 < \alpha < 1$)*.*

Proof. Multiplying the IIKC in eq. (13) by $\boldsymbol{q}'$ and solving for $\boldsymbol{q}' d\boldsymbol{Y}$ yields

$$\boldsymbol{q}' d\boldsymbol{Y} = \frac{\boldsymbol{q}' d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}' d\boldsymbol{T}}{\alpha}, \tag{16}$$

where I used $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ and $\alpha \neq 0$. Note now that iterating on the government's budget constraint and using $\lim_{t\to\infty} B_t/(1+r)^t = 0$ implies $\boldsymbol{q}' d\boldsymbol{T} = \boldsymbol{q}' d\boldsymbol{G}$. Using this in eq. (16) yields $\boldsymbol{q}' d\boldsymbol{Y} = \boldsymbol{q}' d\boldsymbol{G}$, which then implies $\mathcal{M} = 1$ from the definition of the cumulative fiscal multiplier from eq. (15). □

Proposition 2 is a striking result. First, it implies that there is an analytical formula for the cumulative fiscal multiplier. This is rarely the case in a closed economy. Second, the cumulative fiscal multiplier is exactly 1 in any SOE. This is also not the case in a closed economy. In particular, Auclert et al. (2024) shows that it is greater than 1, $\mathcal{M} > 1$, in models with realistic iMPCs like standard heterogeneous agent models.[11] Third, the multiplier in an SOE is exactly one and independent of household behavior. In the closed economy, household behavior is completely central

11. In more elaborate models in Auclert et al. (2024) this is not necessarily the case. Here, the cumulative fiscal multiplier can also be less than 1.

for the fiscal multiplier.[12] Fourth, Proposition 2 implies that there is a discontinuity shown in Figure 1: For any degree of openness $\alpha > 0$, the cumulative fiscal multiplier is 1, even as α becomes closer and closer to 0. However, when α is *exactly* 0, the cumulative fiscal multiplier jumps discontinuously to > 1, as shown in Figure 1.

So why can large MPCs not sustain a boom following a government consumption shock in an SOE, while they do in a closed economy? Note that consumption *can* initially rise in both the closed economy and the SOE. In the closed economy, consumption then returns to steady state, while consumption in the SOE drops below steady state. This is because the SOE runs a current account deficit to finance increased consumption, building a negative net foreign asset position. Eventually, this has to be repaid by cutting back consumption. This drop is large enough to exactly offset the initial rise, such that cumulative consumption is exactly unchanged in present value terms, as shown in Corollary 1, and the cumulative fiscal multiplier is exactly 1. I show an example of this for a particular calibration of a standard HA model in Figure 2.

This mechanism through which fiscal policy "leaks abroad" is also discussed in Aggarwal et al. (2023) and Auclert et al. (2021c), with the latter referring to the effect as "stealing demand from the future".[13] There is no current account deficit in the closed economy, as there are no foreigners to hold the government's debt. Instead, domestic households own the government debt. When the debt is paid back to them, the households eventually spend at home, sustaining the boom.

12. This includes the choice of utility function, the degree of risk aversion, the parametrization of idiosyncratic income shocks, and the MPC.

13. My paper is closely related to Aggarwal et al. (2023), who also study fiscal multipliers in open economies using sequence space methods. A key difference to their paper is that they do not establish my main result in Proposition 2, instead focusing on the cross-country flows of fiscal transfers in a multi-country world economy.

Corollary 1. *The present value consumption change in an SOE following a $d\boldsymbol{G}$ shock is*

$$\sum_{t=0}^{\infty} \frac{dC_t}{(1+r)^t} = 0.$$

Proof. Pre-multiplying $d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T})$ by $\boldsymbol{q}'$ gives $\boldsymbol{q}'d\boldsymbol{C} = \boldsymbol{q}'d\boldsymbol{Y} - \boldsymbol{q}'d\boldsymbol{T} = 0$, using $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$, $\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G}$, and $\mathcal{M} = 1$. □

Note that with a representative agent, consumption is unchanged at all points in time, $dC_t = 0$, implying $dY_t = dG_t$.[14] Here, the path of dT_t does not matter for dC_t. This is Ricardian equivalence. Without Ricardian equivalence, consumption is not unchanged at all points in time, $dC_t \neq 0$, but the present value of consumption is unchanged, as shown in Corollary 1.

3.2 Alternative Model Variations

Having established a unit cumulative fiscal multiplier in the baseline setup, I now consider three possible model variations where Proposition 2 does not hold. Despite this, the conclusion that fiscal policy is less effective in an SOE than in a closed economy still holds in all model variations.

3.2.1 Finite Horizon

So far, I have considered cumulative fiscal multipliers over the infinite horizon. Consider now instead computing the cumulative multiplier up to a finite horizon, T:

$$\mathcal{M}(T) \equiv \frac{\sum_{t=0}^{T} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{T} \frac{dG_t}{(1+r)^t}}. \tag{17}$$

14. See Appendix A.1.1.

In this case, the unit multiplier established in Proposition 2 no longer holds. Figure 3 shows this. In particular, Figure 3 re-creates Figure 1 for different T, including $T \to \infty$ as in Figure 1. The figure shows that the unit multiplier still holds approximately for realistic values of α and values of T which are not too small.

Why is the cumulative fiscal multiplier greater than 1 when truncating the computation of the fiscal multiplier at some $T < \infty$? This is because some of the repayment of debts to foreigners is still ongoing at the point of truncation, i.e. the bust following the boom is not done. Thus, the truncated fiscal multiplier is inflated compared to when it is calculated over an infinite horizon.

3.2.2 Two Large Countries

So far, I have considered an SOE trading with a large economy. I now consider a different model: A 2-country model where the domestic economy is not (necessarily) infinitely small. I then consider the cumulative fiscal multiplier in this setup, which nests the SOE as the size of the domestic economy is 0.

As I am doing away with the SOE assumption, the foreign economy is now affected by the domestic economy. I write the sequence space consumption function in the foreign economy by

$$d\boldsymbol{C}^* = \boldsymbol{M}^* d\boldsymbol{Y}^*,$$

where $\boldsymbol{M}^*$ is the iMPC matrix in the foreign economy. I disregard foreign fiscal policy, as I am interested in the effects of fiscal policy in the domestic economy. Similar to the domestic economy, foreign consumption of home and foreign goods is

$$C^*_{H,t} = \alpha^* C^*_t \quad \text{and} \quad C^*_{F,t} = (1 - \alpha^*) C^*_t.$$

for $\alpha^* \in (0, 1)$, and goods market clearing for foreign output is

$$Y_t^* = C_{F,t}^* + C_{F,t}.$$

I consider for simplicity a domestic economy steady state with zero net exports and government consumption. Then $C_{H,ss}^* = C_{F,ss}$, and inserting this into goods market clearing yields $\alpha Y_{ss}^* = \alpha Y$ due to $Y_{ss} = C_{ss}$ and $Y_{ss}^* = C_{ss}^*$. Solving for α^* then gives $\alpha^* = \alpha Y_{ss} / Y_{ss}^*$. This is simply an accounting equation that ensures that flows across countries are consistent. Linearizing and stacking then gives the *multi-country IIKC*:

$$\begin{pmatrix} d\boldsymbol{Y} \\ d\boldsymbol{Y}^* \end{pmatrix} = \begin{pmatrix} (1-\alpha)\boldsymbol{M} & \alpha \frac{Y_{ss}}{Y_{ss}^*} \boldsymbol{M}^* \\ \alpha \boldsymbol{M} & \left(1 - \alpha \frac{Y_{ss}}{Y_{ss}^*}\right) \boldsymbol{M}^* \end{pmatrix} \begin{pmatrix} d\boldsymbol{Y} - d\boldsymbol{T} \\ d\boldsymbol{Y}^* \end{pmatrix} + \begin{pmatrix} d\boldsymbol{G} \\ \boldsymbol{0} \end{pmatrix}. \tag{18}$$

I am now ready to study the cumulative fiscal multiplier in this more general model. I start by noting that the cumulative fiscal multiplier is 1, $\mathcal{M} = 1$, when the domestic economy is infinitesimal, i.e. Y_{ss}. This is simply the case of Proposition 2.

The more interesting case is $Y_{ss} > 0$. Here—analogously to the closed economy—I cannot use the Keynesian cross to derive the cumulative fiscal multiplier. Instead, I take the numerical approach of simulating the model. Figure 4 shows the cumulative fiscal multipliers for different relative sizes of the economies, Y_{ss}/Y_{ss}^*. The figure shows that the cumulative multiplier is not exactly 1 when the economy is not fully infinitesimal, but instead small, i.e. $Y_{ss} > 0$. However, the multiplier remains very close to 1 for any value of Y_{ss}/Y_{ss}^* which can reasonably be called small. As an example, note that Italy—A G7 country and among the 10 largest economies in the world—has a GDP share of around 2% of the world's GDP, i.e. $Y_{ss}/Y_{ss}^* < 3\%$ which is the largest value shown in Figure 4.

This shows that while the assumption of the SOE being infinitesimal *is* required for $\mathcal{M} = 1$, the cumulative fiscal multiplier is still very close to 1 if the SOE is small but not infinitesimal. Furthermore, it appears that the cumulative fiscal multiplier is still discontinuous in the degree of openness even when $Y_{ss}/Y^*_{ss} > 0$.[15]

Why is the cumulative fiscal multiplier (slightly) larger when the domestic economy is not infinitesimal? This is because some of the consumption that goes abroad is no longer lost, but instead returns as foreign households increase consumption. And this effect is stronger the larger the domestic economy is relative to the foreign economy. Perhaps more surprisingly, why does the cumulative fiscal multiplier not depend on the degree of openness? While it is true that a larger degree of openness implies that more consumption is lost abroad, it also implies that foreign households demand more domestic goods, i.e. the degree of openness abroad is also larger due to the positive relationship between α and α^*.

3.2.3 Active Monetary Policy

So far, the nominal interest rate has responded one-to-one with inflation such that the real interest rate is fixed. This is a natural starting point as it strikes "a middle ground between loose policy (like at the zero lower bound) and tight policy (like with an active Taylor rule)", as argued in Auclert et al. (2024). I now relax this assumption and consider active monetary policy according to the following Taylor rule:

$$i_t = i_{ss} + \phi_\pi \pi_t, \tag{19}$$

where $i_t = (1 + r_t)(1 + \pi_{t+1}) - 1$ is the nominal interest rate and π_t is inflation.[16] In this case, the cumulative fiscal multiplier is as follows.

15. I note that this clearly appears to be the case from Figure 4 but I have not formally proven it.

16. See the details in Appendix A.1.3.

Proposition 3. *The cumulative fiscal multiplier is*

$$\mathcal{M} = 1 + \underbrace{\frac{1-\alpha}{\alpha}\frac{A_{ss}}{1+r}\frac{\boldsymbol{q}'d\boldsymbol{r}}{\boldsymbol{q}'d\boldsymbol{G}}}_{\textit{Wealth effect}} + \underbrace{\left[\frac{2-\alpha}{1-\alpha}\eta C_{ss} - 1\right]\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}}}_{\textit{Exchange rate}} + \underbrace{G_{ss}\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} - \frac{1-\alpha}{\alpha}PD_{ss}\sum_{t=0}^{\infty} dq_t}_{\textit{Financing cost}},$$

where dq_t is a perturbation of $q_t = (1+r_0)^{-1}\dots(1+r_{t-1})^{-1}$ around $(1+r)^{-t}$.

Proof. The proof follows the proof of Proposition 2 using a generalized IIKC with a varying real interest rate. See Appendix A.2 for details. □

In the case with a constant real interest rate, $d\boldsymbol{r} = d\boldsymbol{Q} = 0$ and $dq_t = 0$, so $\mathcal{M} = 1$. With active monetary policy, the real interest rate appreciates due to higher domestic demand.[17]. I consider the case of a fixed exchange rate instead of the Taylor rule in Appendix A.1.4, which yields similar results. The higher real interest rate creates the following new channels:

1. **Wealth effect.** A higher real interest rate implies that households are wealthier, stimulating consumption and boosting output in general equilibrium.

2. **Exchange rate.** A higher real interest rate appreciates the real exchange rate by capital inflow. The appreciation makes consumers substitute away from domestic goods, lowering output. It also makes domestic consumers richer in real terms (Auclert et al. 2021c), boosting consumption and output. I note that both (i) the sign of the response of the real exchange rate and (ii) which of the two channels dominates depends on the calibration.

3. **Financing cost.** The higher real interest rate makes debt more expensive. The appreciated real exchange rate makes government consumption more

17. See Figure A.2 in Appendix A.1.4

expensive. Both imply that the government has to increase taxes more to finance government consumption, lowering consumption and output.

Both the wealth effect and the financing cost are unimportant in the calibration of Appendix A.1.5. Thus, movements in the real exchange rate are what matter. The empirical literature is not clear on what happens with the real exchange rate following a fiscal policy shock. Some contributions find that the real exchange rate depreciates following fiscal stimulus (Kim and Roubini 2008, Monacelli and Perotti 2010, Ravn et al. 2012, and Kim 2015), while others find an appreciation (Miyamoto et al. 2019, Ferrara et al. 2021, and Born et al. 2024).

Proposition 3 implies that the cumulative fiscal multiplier can be different than 1 when monetary policy is active. But what is the fiscal multiplier and how does the conduct of monetary policy affect it? In Figure 5, I show the cumulative fiscal multiplier as a function of the degree of openness for different calibrations of the reaction to inflation in the Taylor rule, ϕ_π, and the NKWPC slope, κ. First, the figure shows that the cumulative fiscal multiplier is no longer 1, but can be both above or below 1. Second, the figure shows that a more hawkish central bank (higher ϕ_π) and a more steep NKWPC (higher κ) are associated with a lower cumulative fiscal multiplier. Third, the figure shows that the cumulative fiscal multiplier tends to decrease in the degree of openness for most calibrations.

3.3 Self-Financing Fiscal Deficits

3.3.1 The Definition of Self-Financing

Having studied fiscal multipliers in the SOE, I now turn to the degree of self-financing of fiscal deficits. This is motivated by Angeletos et al. (2024), who show that fiscal deficits can be fully self-financing, i.e. that the government never actively has to raise taxes to pay back debts. Instead, they show that the boom created by the fiscal stimulus creates a large enough increase in labor tax revenue to pay back all debts due to the fiscal stimulus (100% self-financing). However, their result is in a *closed economy*. Given that I have shown that the boom in a *small open economy* is smaller, what does this mean for the degree of self-financing?

To study this, I introduce a labor tax as in Angeletos et al. (2024) to my model from Section 2 to have a possibility of self-financing. In this case, the government's primary deficit is[18]

$$\mathrm{PD}_t = \frac{P_{H,t}}{P_t} G_t - T_t - \tau Y_t,$$

and the aggregate household budget constraint is

$$C_t + A_t = (1+r)A_{t-1} + (1-\tau)Y_t - T_t.$$

Iterating on the government's budget constraint yields

$$\frac{B_t}{(1+r)^t} = (1+r)B_{ss} + \sum_{s=0}^{t} \frac{G_s - T_s}{(1+r)^s} - \tau \sum_{s=0}^{t} \frac{Y_s}{(1+r)^s}. \tag{20}$$

The $G_s - T_s$ term is controlled directly by the government, while the τY_s term reflects self-financing: Deficits create a boom that increases labor income and hence labor taxes, reducing the deficit. Following Angeletos et al. (2024), I define the degree of self-financing in the limit as $t \to \infty$ by

$$\nu \equiv \frac{\tau \sum_{t=0}^{\infty} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{dG_t - dT_t}{(1+r)^t}} = \frac{\tau \boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T})}. \tag{21}$$

This object measures the receipts from labor taxes relative to the fiscal deficit

18. Using the fact that $w_t N_t = Y_t$.

from increased government consumption and/or lump-sum taxes. If the labor taxes raised are enough to exactly cover the deficit in a present-value sense, the degree of self-financing is 1, i.e. fiscal deficits are self-financing. In this case, both the present value of primary deficits and the present value of government bonds are zero as $t \to \infty$, which I assumed directly in Section 2. If instead labor taxes do not cover the deficit, the degree of self-financing is less than 1.

3.3.2 Self-Financing In Closed and Open Economies

With the definition of self-financing in mind, I now provide some results on the financing of deficits in closed and open economies.

Proposition 4. *Consider two fiscal policy shocks: (1) A government consumption shock, $d\boldsymbol{G} \neq \boldsymbol{0}$, with fixed lump-sum taxes, $d\boldsymbol{T} = \boldsymbol{0}$, or (2) a lump-sum tax shock, $d\boldsymbol{T} \neq \boldsymbol{0}$, with fixed government consumption, $d\boldsymbol{G} = \boldsymbol{0}$. Define their degrees of self-financing as ν_G and ν_T.*

- *In a closed economy, a fiscal deficit is completely self-financing, i.e. $\nu_G = \nu_T = 1$.*
- *In an SOE, $\alpha > 0$, a fiscal deficit is not self-financing, i.e. $\nu_G < 1$ and $\nu_T < 1$:*

$$\nu_T = \frac{(1-\alpha)\tau}{(1-\alpha)\tau + \alpha}, \quad \textit{and} \quad \nu_G = \frac{\tau}{(1-\alpha)\tau + \alpha}. \tag{22}$$

Proof. The IIKC is now

$$d\boldsymbol{Y} = (1-\alpha)(1-\tau)\boldsymbol{M}d\boldsymbol{Y} - (1-\alpha)Md\boldsymbol{T} + d\boldsymbol{G}.$$

Pre-multiplying by $\boldsymbol{q}'$ and solving for $\boldsymbol{q}'d\boldsymbol{Y}$ then gives

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{\boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T}}{(1-\alpha)\tau + \alpha}.$$

Observing the two cases of $d\boldsymbol{G} = 0$ and $d\boldsymbol{T} = 0$ each and using the definition of the degree of self-financing in eq. (21) then gives eq. (22). The closed economy result is obtained for $\alpha = 0$. $\square$

The closed economy result in the first bullet is the main result in Angeletos et al. (2024) when financing is delayed forever. The result implies that the government does not have to reduce transfers or change the tax rate to finance government consumption. Instead, the tax receipts from the boom are enough to cover the deficit.

The SOE result in the second bullet implies that this result does not extend to an SOE. In this case, the degree of self-financing is less than perfect. The intuition is similar to Proposition 2: Fiscal policy creates less of a boom when the economy is open. However, in contrast with Proposition 2, the degree of self-financing is not discontinuous at $\alpha = 0$, but converges to $\nu = 1$ as α approaches 0 from above.

The result for the SOE is striking because it contains simple, intuitive analytical formulae for the degrees of self-financing, something that does not exist for the closed economy. Intuitively, the degree of self-financing is increasing in the tax rate, τ, simply because more taxes are paid. Additionally, the degree of self-financing is decreasing in the degree of openness, α, because more consumption is "lost abroad".

In the closed economy limit of $\alpha = 0$, there is perfect self-financing: $\nu_G = \nu_T = 1$. How far does the SOE deviate from this closed economy benchmark? If, for instance, $\tau = 0.3$ (as in Angeletos et al. 2024) and $\alpha = 0.4$, $\nu_G = 0.52$ and $\nu_T = 0.31$, i.e. there is half or less self-financing.

Lastly, it is worth considering why the degree of self-financing differs for government consumption and taxes when the economy is open ($\nu_G \neq \nu_T$). In particular, why is the tax multiplier lower by a factor $1 - \alpha$, i.e. $\nu_T = (1 - \alpha)\nu_G$? This is because a share $1 - \alpha$ of transfers are spent abroad by domestic households, while the government only buys domestic goods. This is not the case in a closed economy, where the degrees of self-financing are the same: $\nu_G = \nu_T = 1$.

4 Conclusion

I study fiscal policy by re-visiting the closed economy results of Auclert et al. (2024) and Angeletos et al. (2024) in an SOE. I show that fiscal policy is much less effective at stimulating the domestic economy in an SOE than in a closed economy even with realistic intertemporal marginal propensities to consume: Cumulative consumption is unchanged in present value terms, the cumulative fiscal multiplier is exactly 1, and fiscal deficits are not fully self-financing.

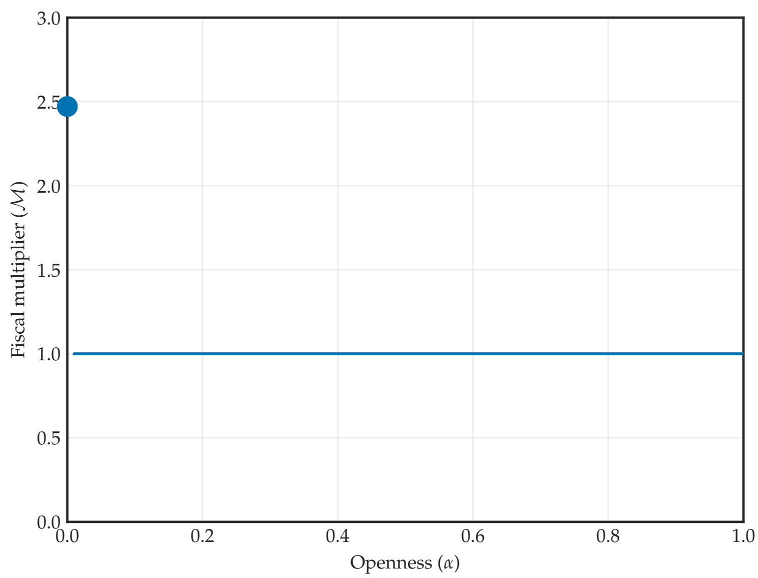

Figure 1: The Cumulative Fiscal Multiplier for Different Degrees of Openness

Note: The figure shows the cumulative fiscal multiplier for different values of α. The cumulative fiscal multiplier in the closed economy is from the HA model in Appendix A.1.2 using the calibration from Appendix A.1.5 with $\alpha = 0$.

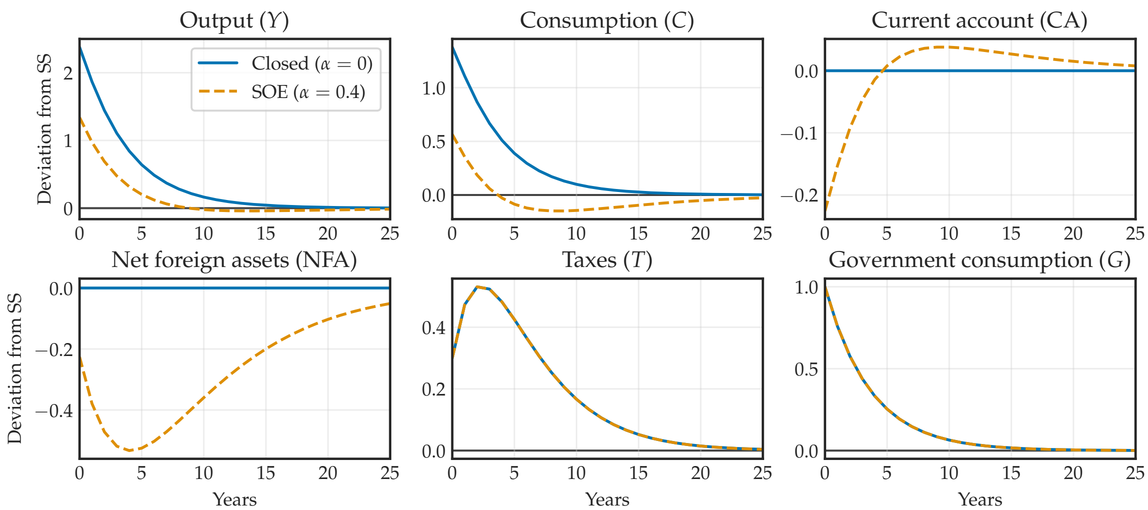

Figure 2: IRFs to a Government Consumption Shock

Note: The figure shows impulse response functions (IRFs), i.e. $100 \cdot d\boldsymbol{X}$, for various $\boldsymbol{X}$ from the model with an HA household side using the calibration from Appendix A.1.5.

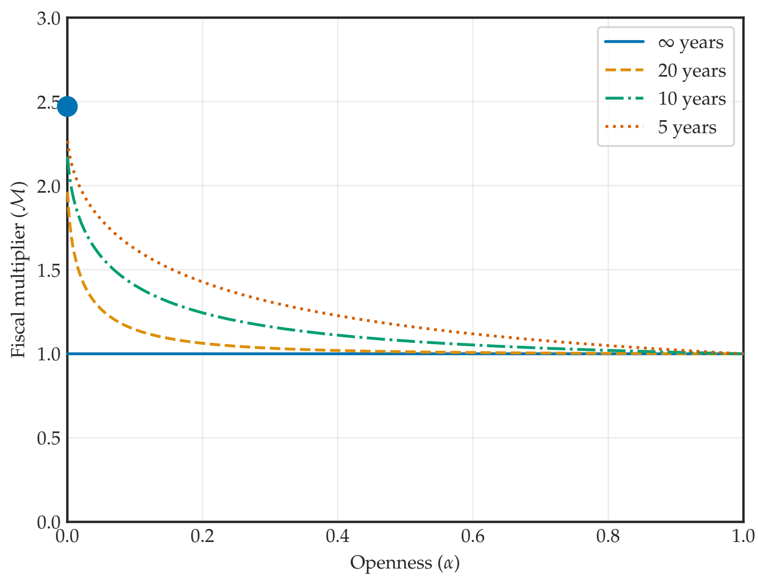

Figure 3: The Cumulative Fiscal Multiplier at Different Horizons

Note: The figure shows the cumulative fiscal multiplier from eq. (17) for different values of α and different horizons, T. The fiscal multipliers are computed using the model with an HA household side using the calibration from Appendix A.1.5.

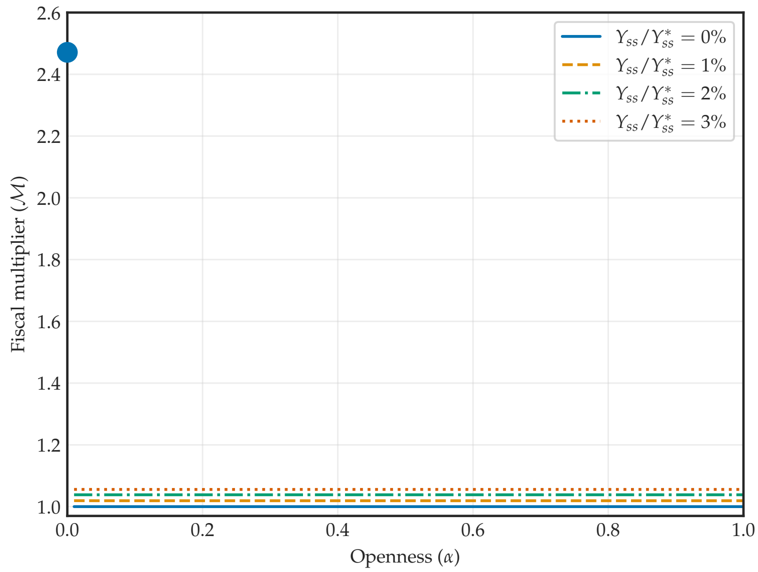

Figure 4: The Cumulative Fiscal Multiplier for Two Large Economies

Note: The figure shows the cumulative fiscal multiplier for different values of α and different Y_{ss}/Y^*_{ss}. The fiscal multipliers are computed using the model with an HA household side using the calibration from Appendix A.1.5.

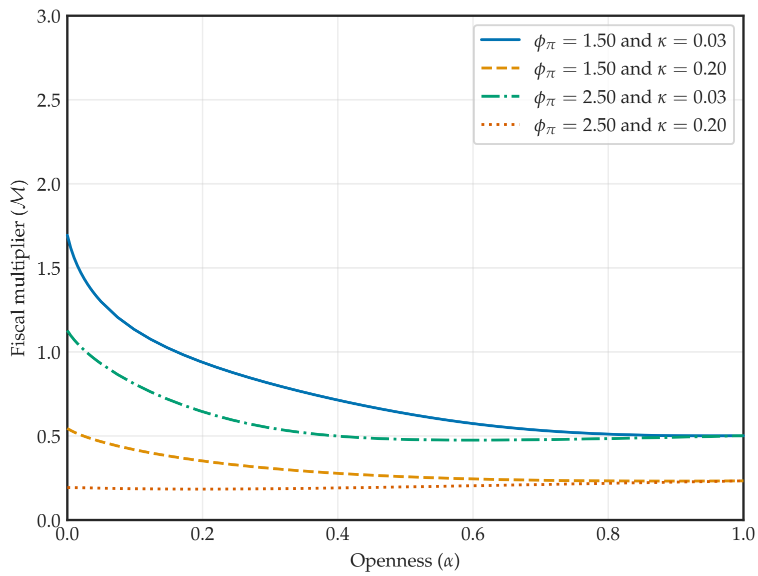

Figure 5: The Cumulative Fiscal Multiplier with Active Monetary Policy

Note: The figure shows $\mathcal{M}$ for different values of ϕ_π and κ in the model from Appendix A.1.3.

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Appendix:

Fiscal Policy in Small Open Economies: The International Intertemporal Keynesian Cross

Jacob Marott Sundram

A.1 Model Examples and Details

A.1.1 A Representative Agent Model

The representative agent solves

$$V_t(A_{t-1}) = \max_{C_t, A_t} u(C_t) + \beta \mathbb{E}_t \left[V_{t+1}(A_t) \right], \tag{23}$$

s.t.

$$C_t + A_t = (1 + r_{t-1})A_{t-1} + Z_t, \tag{24}$$

$$\lim_{t \to \infty} q_t A_t = 0. \tag{25}$$

Solving this yields a standard Euler equation, which pins down consumption growth:

$$C_{t+1} = \left[\beta(1 + r_t)\right]^{\frac{1}{\sigma}} C_t, \tag{26}$$

for $t = 0, 1, \dots$ With $r_t = r$, this implies $C_t = C_{t+1}$. Using this with the transversality condition in eq. (25) and budget constraint in eq. (24) gives

$$C_0 = (1 - \beta) \left[(1 + r_{ss})A_{ss} + \sum_{t=0}^{\infty} q_t Z_t \right]. \tag{27}$$

Since this model satisfies the standard aggregate budget constraint and transversality condition, it follows by Proposition 2 that $\mathcal{M} = 1$. This implies that $\boldsymbol{q}' d\boldsymbol{Z} = 0$, so $dC_0 = 0$ by eq. (27). Since the real interest rate is fixed, it follows from the Euler equation in eq. (26) that $dC_t = 0$ for all t, which finally implies $dY_t = dG_t$. Figure A.1 shows this. This is the same result shown in Woodford (2011). Thus, Proposition 2 can be seen as generalizing the result in Woodford (2011) through the lens of cumulative multipliers to *any household side* while also highlighting that $dY_t = dG_t$ is a special case of the representative agent model.

A.1.2 A Heterogeneous Agents Model

The household side is characterized by a standard incomplete markets household problem (Aiyagari 1994, Bewley 1979, Huggett 1993, Imrohoroğlu 1989). There is a continuum of households. Each household with assets a, and idiosyncratic earnings e chooses consumption c, and next-period assets a' (where primes denote variables in the next period). They choose this to solve the following dynamic problem:

$$V_t(a,e) = \max_{c,a'} u(c) + \beta \mathbb{E}_t \left[V_{t+1}(a',e') \right],$$

$$\text{s.t.}$$

$$c + a' = (1 + r_{t-1})a + Z_t \frac{e^{1-\theta}}{\mathbb{E}\left[e^{1-\theta}\right]},$$

$$a' \geq 0.$$

The aggregate variables are post-tax labor income, Z_t, and the ex-post real return on assets, r_{t-1}. u is the instantaneous utility of consumption and the disutility of labor supply, respectively. $\theta \in [0,1]$ controls tax progressivity following Heathcote et al. (2017). As is standard in the HANK literature (Auclert et al. 2021a), the labor union chooses labor supply, N_t, see Appendix A.1.3.

Log idiosyncratic earnings, $\log e$, follows an AR(1) process with persistence ρ_e and variance σ_e^2. I discretize this as a Markov chain normalized such that $\mathbb{E}[e] = 1$. Utility of consumption and disutility of labor follow standard functional forms,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{and} \quad v(n) = \Gamma n^{1+\frac{1}{\phi}},$$

where $\sigma > 0$ is the inverse elasticity of intertemporal substitution, $\phi > 0$ is the Frisch elasticity of labor supply, and $\Gamma > 0$ is a normalization constant.

A.1.3 Active Monetary Policy

In this section, I present the model from Section 2.1 augmented with active monetary policy. This requires describing the public sector and the labor union in some more detail.

The government sets a labor tax rate, τ, and uses lump-sum transfers to balance the budget according to the following rule from Auclert et al. (2024):

$$dB_t = \phi_B(dB_{t-1} + dG_t).$$

The central bank follows a monetary policy rule in CPI inflation, $\pi_t = P_t/P_{t-1} - 1$:

$$i_t = r_{ss} + \phi_\pi \pi_t.$$

The nominal and real interest rate is connected through a Fisher equation:

$$1 + i_t = (1 + r_t)(1 + \pi_{t+1}).$$

Labor supply is set by unions subject to quadratic costs of adjusting the nominal wage. This is exactly as in Auclert et al. (2024), who show that the problem yields the following new-Keynesian wage Phillips curve (NKWPC):

$$\pi_t^w(1+\pi_t^w) = \kappa^w \left(\frac{\Gamma N_t^{1/\phi}}{(C_t^v)^{-\sigma}(1-\theta)Z_t/N_t} - 1 \right) + \beta \pi_{t+1}^w (1+\pi_{t+1}^w),$$

where Γ is chosen such that the NKWPC holds in steady state, $\pi_t^w = W_t/W_{t-1} - 1$ is wage inflation, and C_t^v is a virtual consumption aggregate given by

$$C^v \equiv \left\{ \mathbb{E}\left[\frac{e_{i,t}^{1-\theta}}{\mathbb{E}(e_{i,t}^{1-\theta})} c_{i,t}^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}},$$

where the expectation is taken across the distribution of households. With a fixed real interest rate, this NKWPC does not matter for real outcomes.

A.1.4 A Fixed Exchange Rate

I consider replacing the monetary policy rule in eq. (19) with a fixed exchange rate:

$$E_t = E_{ss}.$$

Figure A.2 shows IRFs for both active monetary policy and a fixed exchange rate.

A.1.5 Calibration

I simulate the model for some of the figures in the main text. To do this, I use the calibration from Auclert et al. (2024).[19] Two parameters related to openness are not present in their model: α and η. For α, I follow Auclert et al. (2021c) and set $\alpha = 0.4$. For η, I choose $\eta = 1$. I note that η only matters when monetary policy is active.

19. They consider many different values for ϕ_B, so I choose $\phi_B = 0.7$. For larger values, the cumulative fiscal multiplier becomes large in the closed economy.

A.2 Proof of Proposition 3

A.2.1 Domestic Consumption of Domestic Goods

Linearizing CPI in eq. (7) yields

$$dP_t = \alpha dP_{F,t} + (1-\alpha)dP_{H,t}. \tag{28}$$

Similarly, linearizing the real exchange rate in eq. (11) implies that

$$dQ_t = dP_{F,t} - dP^*_{F,t} + dP^*_t - dP_t, \tag{29}$$

using $dE_t = dP_{F,t} - dP^*_{F,t}$ linearized. Inserting $dP^*_{F,t} = dP^*_t = 0$ yields $dQ_t = dP_{F,t} - dP_t$. Using this in the linearized CPI in eq. (28) and solving for $dP_{H,t} - dP_t$, I find that

$$dP_{H,t} - dP_t = -\frac{\alpha}{1-\alpha} dQ_t. \tag{30}$$

Using this in eq. (6) linearized yields:

$$dC_{H,t} = (1-\alpha)dC_t + \eta\alpha C_{ss} dQ_t. \tag{31}$$

A.2.2 Foreign Consumption of Domestic Goods

Linearizing the price of home goods abroad gives

$$dP^*_{H,t} = dP_{H,t} - dE_t.$$

Subtracting eq. (29) solved for dP^*_t gives

$$dP^*_{H,t} - dP^*_t = dP_{H,t} - dP_t - dQ_t.$$

Inserting eq. (30) gives

$$dP^*_{H,t} - dP^*_t = -\frac{1}{1-\alpha} dQ_t.$$

Using this in eq. (10) linearized yields:

$$dC^*_{H,t} = \alpha dC^*_t + \frac{\alpha}{1-\alpha}\eta C^*_{ss} dQ_t. \tag{32}$$

A.2.3 Jacobian with Respect to Real Labor Income

Lemma 1 gives that $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ in the case with a fixed real interest rate. I now show that this extends to the case with a time-varying real interest rate. I start by iterating on the aggregate budget constraint in eq. (1) to get that

$$q_t A_t = (1 + r_{ss})A_{ss} + \sum_{s=0}^{t} q_s (Z_s - C_s),$$

where

$$q_t \equiv (1 + r_0)^{-1}(1 + r_1)^{-1} \ldots (1 + r_{t-1})^{-1}, \quad \text{for } t = 1, 2, \ldots \tag{33}$$

and $q_0 \equiv 1$. Using the transversality condition, $\lim_{t\to\infty} q_t A_t = 0$, gives[20]

$$\sum_{t=0}^{\infty} q_t C_t = (1 + r_{ss})A_{ss} + \sum_{t=0}^{\infty} q_t Z_t. \tag{34}$$

20. This is the transversality with a time-varying real interest rate. It collapses to the one in eq. (2) when $r_t = r$.

Taking the derivative w.r.t. Z_s yields

$$\sum_{t=0}^{\infty} q_t M_{t,s} = q_s.$$

Evaluating in the steady state then gives

$$\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^t} = \frac{1}{(1+r)^s}.$$

This in turn implies that $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$, i.e. the present-value MPC is 1.

A.2.4 Jacobian with Respect to Returns

Taking the derivative of eq. (34) w.r.t. r_s yields:

$$\sum_{t=0}^{\infty} q_t M^r_{t,s} + \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s} C_t = \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s} Z_t,$$

where $M^r_{t,s} = \partial C_t / \partial r_s$ is element (t,s) of $\boldsymbol{M^r}$. Evaluating in the steady state gives:

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r_{ss})^t} + C_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss} = Z_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss},$$

where "$|_{ss}$" denotes evaluation in the steady state. Note that evaluating the aggregate budget constraint from eq. (1) in the steady state yields $C_{ss} - Z_{ss} = r_{ss} A_{ss}$ such that

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r)^t} = -r_{ss} A_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss}. \tag{35}$$

By differentiating the definition of q_t in eq. (33), I find that

$$\frac{\partial q_t}{\partial r_s}\Big|_{ss} = -\frac{1}{(1+r_{ss})^{t+1}}, \quad \text{for } s = 0, 1, \ldots, t-1$$
$$\frac{\partial q_t}{\partial r_s}\Big|_{ss} = 0, \quad \text{for } s = t, t+1, \ldots$$

Evaluating this in the steady state gives

$$\sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss} = -\sum_{t=s}^{\infty} \frac{1}{(1+r_{ss})^{t+2}} = -\frac{1}{r_{ss}} \frac{1}{(1+r_{ss})^{s+1}},$$

Inserting this in eq. (35) gives

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r)^t} = \frac{A_{ss}}{1+r_{ss}} \frac{1}{(1+r_{ss})^s},$$

which finally implies

$$\boldsymbol{q}'\boldsymbol{M}^{\boldsymbol{r}} = \frac{A_{ss}}{1+r_{ss}}\boldsymbol{q}'. \tag{36}$$

A.2.5 Main Proof

The linearized consumption function is

$$d\boldsymbol{C} = \boldsymbol{M}d\boldsymbol{Z} + \boldsymbol{M}^{\boldsymbol{r}}d\boldsymbol{r} \tag{37}$$

with Jacobians

$$\boldsymbol{M} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{Z}}, \quad \boldsymbol{M}^{\boldsymbol{r}} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}}.$$

Insert now $W_t = P_{H,t}$ into the definition of labor income to get that

$$Z_t = \frac{P_{H,t}}{P_t} Y_t - T_t.$$

Taking a first-order approximation of this yields

$$dZ_t = dY_t - dT_t + dP_{H,t} - dP_t = dY_t - \frac{\alpha}{1-\alpha} dQ_t,$$

where I used eq. (30) in the second equality. Inserting this into the linearized consumption function in eq. (37), it follows that

$$d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) - \frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q} + \boldsymbol{M}^{r}d\boldsymbol{r}. \tag{38}$$

I use this shortly. To do this, turn next to linearized goods market clearing:

$$d\boldsymbol{Y} = d\boldsymbol{C_H} + d\boldsymbol{C_H^*} + d\boldsymbol{G}.$$

Insert eq. (31) and eq. (32) and gathering terms yields:

$$d\boldsymbol{Y} = (1-\alpha)d\boldsymbol{C} + \left[\eta\alpha C_{ss} + \eta\frac{\alpha}{1-\alpha}C_{ss}^*\right]d\boldsymbol{Q} + d\boldsymbol{G},$$

using $d\boldsymbol{C}^* = 0$. Inserting eq. (38) yields

$$d\boldsymbol{Y} = (1-\alpha)\left[\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) - \frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q} + \boldsymbol{M}^{r}d\boldsymbol{r}\right] + \left[\eta\alpha + \eta\frac{\alpha}{1-\alpha}\right]C_{ss}d\boldsymbol{Q} + d\boldsymbol{G}$$

using $C_{ss} = C_{ss}^*$. Isolating $d\boldsymbol{Y}$ then gives

$$\left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right]d\boldsymbol{Y} = (1-\alpha)\left[\boldsymbol{M}^{r}d\boldsymbol{r} - \boldsymbol{M}d\boldsymbol{T}\right] + \left[\frac{2-\alpha}{1-\alpha}\alpha\eta C_{ss}\boldsymbol{I} - \alpha\boldsymbol{M}\right]d\boldsymbol{Q} + d\boldsymbol{G}.$$

Pre-multiplying by $\boldsymbol{q}'$ gives:

$$\alpha\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\frac{A_{ss}}{1+r_{ss}}\boldsymbol{q}'d\boldsymbol{r} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T} + \left[\frac{2-\alpha}{1-\alpha}\alpha\eta C_{ss} - \alpha\right]\boldsymbol{q}'d\boldsymbol{Q} + \boldsymbol{q}'d\boldsymbol{G}, \tag{39}$$

where I used eq. (36). Note next that $\boldsymbol{q}'d\boldsymbol{G} \neq \boldsymbol{q}'d\boldsymbol{T}$ when r_t is time-varying. Iterating on the government's budget constraint and using $\lim_{t\to\infty} B_t/(1+r)^t$ yields

$$\sum_{t=0}^{\infty} q_t \mathrm{PD}_t + (1+r)B_{ss} = 0.$$

Taking a first order approximation and using $\mathrm{PD}_t = (P_{H,t}/P_t)G_t - T_t$ gives

$$\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G} + G_{ss}\boldsymbol{q}'(d\boldsymbol{P_H} - d\boldsymbol{P}) + \mathrm{PD}_{ss}\sum_{t=0}^{\infty} dq_t,$$

where dq_t is a perturbation of q_t around $(1+r)^{-t}$. Note that the case of constant real interest rate gives $dq_t = 0$ and $d\boldsymbol{P_H} = d\boldsymbol{P} = 0$, so $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$. Using eq. (30) gives

$$\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G} - G_{ss}\frac{\alpha}{1-\alpha}\boldsymbol{q}'d\boldsymbol{Q} + \mathrm{PD}_{ss}\sum_{t=0}^{\infty} dq_t. \tag{40}$$

Inserting eq. (40) into eq. (39) yields

$$\begin{aligned}\alpha\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\frac{A_{ss}}{1+r_{ss}}\boldsymbol{q}'d\boldsymbol{r} + \left[\frac{2-\alpha}{1-\alpha}\alpha\eta C_{ss} - \alpha\right]\boldsymbol{q}'d\boldsymbol{Q} + \alpha\boldsymbol{q}'d\boldsymbol{G}\\ + G_{ss}\alpha\boldsymbol{q}'d\boldsymbol{Q} - (1-\alpha)\mathrm{PD}_{ss}\sum_{t=0}^{\infty} dq_t,\end{aligned}$$

where I used eq. (36). Dividing by $\alpha\boldsymbol{q}'d\boldsymbol{G}$ finally gives:

$$\mathcal{M} = 1 + \frac{1-\alpha}{\alpha}\frac{A_{ss}}{1+r}\frac{\boldsymbol{q}'d\boldsymbol{r}}{\boldsymbol{q}'d\boldsymbol{G}} + \left[\frac{2-\alpha}{1-\alpha}\eta C_{ss} - 1\right]\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} + G_{ss}\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} - \frac{1-\alpha}{\alpha}\mathrm{PD}_{ss}\sum_{t=0}^{\infty} dq_t.$$

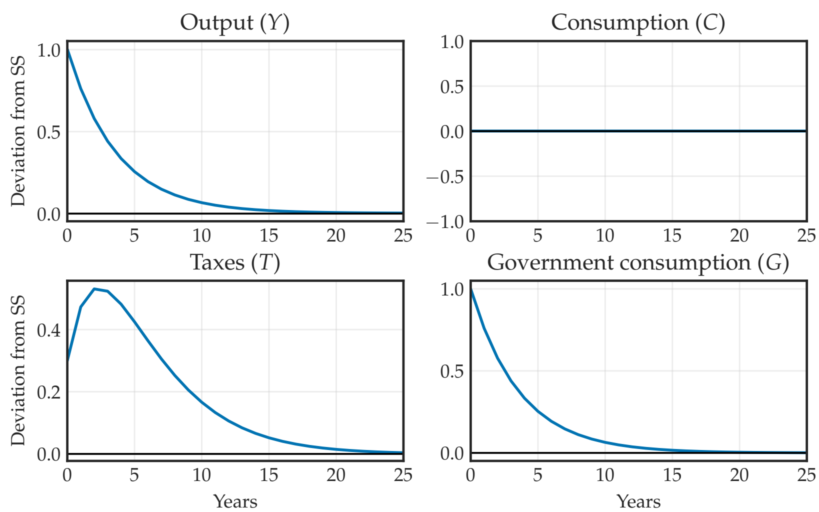

Figure A.1: IRFs to a Government Consumption Shock with a Representative Agent

Note: The figure shows $100 \cdot d\boldsymbol{X}$ for various $\boldsymbol{X}$ from the representative agent model using the calibration from Appendix A.1.5.

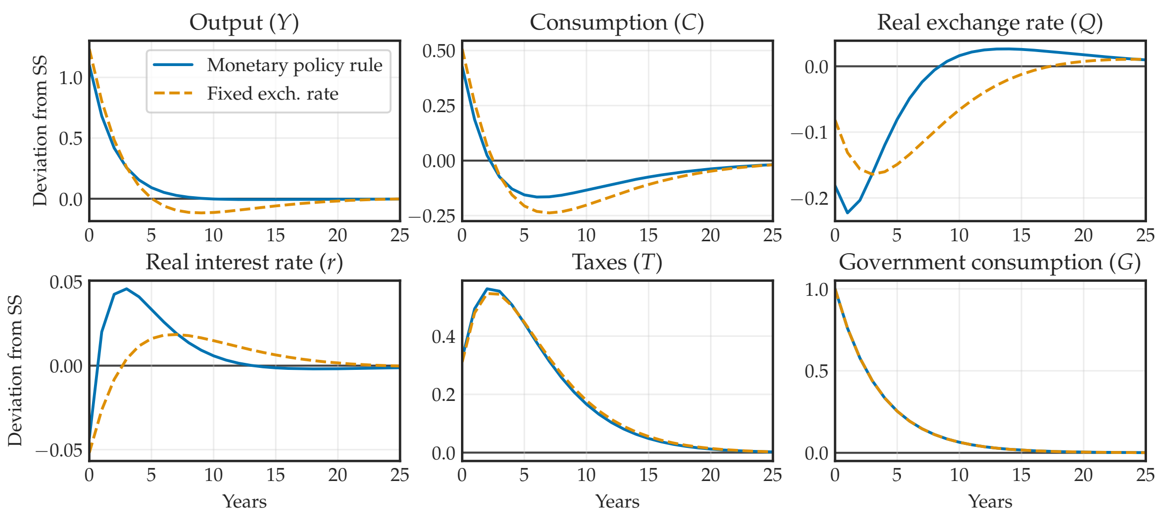

Figure A.2: IRFs to a Government Consumption Shock with Active Monetary Policy and a Fixed Exchange Rate

Note: The figure shows $100 \cdot d\boldsymbol{X}$ for various $\boldsymbol{X}$ from the model in Appendix A.1.3 with both a monetary policy rule and a fixed exchange rate. The tax rule satisfies $dT_t = \rho_B(dB_{t-1} + dG_t)$ as in Auclert et al. (2024).

Parameter	σ	φ	β	ρ_e	σ_e	θ	r
Value	1	1	0.871	0.91	0.92	0.181	0.05
Parameter	B_{ss}	G_{ss}	ϕ_B	α	η	dG_0	ρ_G
Value	0.21	0.20	0.7	0.4	1.0	0.01	0.76

Table A.1: Calibration

Note: This table shows the calibration of the baseline SOE model.